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This reports on the work done on stochastic comparisons of semimartingale Markov processes, opportunistic replacement policies, stochastic flows, and the book project on probability theory and stochastic processes.

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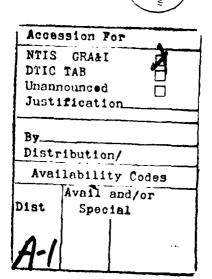
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MARKOV PROCESSES
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Work under this grant during 1987-88 resulted in two papers, six finished chapters on the book project, three extra chapters in draft form, and substantial progress on a big research project on stochastic flows. Enclosed are copies of the two papers and six book chapters. The other chapters are not typed yet. The remaining work on stochastic flows will have to wait some more resolution before we can cut it into research papers (I expect three or four papers to come out of it). The following is a brief description of these items.

1. Stochastic comparison of Semimartingale Hunt Processes

We had shown some time ago* that any Hunt process on R^n that is also a semimartingale can be obtained from some Wiener processes and a Poisson random measure through well understood operations. One end result is that the probability law of such a process is described completely by four deterministic functions. These functions are easy to interpret and yield a convenient picture of the intrinsic features (or, rather, the dynamics) of the process under study.

The present work provides stochastic comparisons of semimartingale Hunt processes in terms of those four functions. The results clarify and extend almost all the known results on stochastic comparisons of Markov processes.

2. Opportunistic Replacement

Optimal opportunistic replacement policies are generally obtained via dynamic programming. Such methods are difficult to implement because the size of the state space gets too large. We are using analytic techniques, based on Markov renewal processes (with arbitrary state spaces), to obtain the optimal policies. The variation of the optimal cost as a function of various

*E. CINLAR and J. JACOD (1981). Representation of semimartingale Markov processes in terms of Wiener processes and Poisson random measures. *Seminar on Stochastic Processes* 1981, 159-242.

costs is studied numerically.

3. Book Project

This is to be a book on probability theory and stochastic processes. The first six chapters are in (nearly) final form. These are enclosed. This project has consumed more than 60% of our time. Three extra chapters, on Lévy processes, on stochastic integration, and on stochastic differential equations, are nearly complete but not typed yet. There remains three chapters to complete -- two on Markov processes, and one on the connections between Markov processes and stochastic calculus.

4. Stochastic Flows

A stochastic flow is a family of random variables $F_{st}x$ where $o \leq s < t$ are thought as times and $x \in \mathbb{R}^n$ as a spatial variable. We think of $F_{st}x$ as the location at time t of a virtual particle whose location was x at time s. The simpler examples come from the field of stochastic differential equations where $F_{st}x$ is the solution of

$$dy(t) = u(y(t))dt + v(y(t))dW_t , \quad y(s) = x .$$

More generally, it is possible to write $F_{st}x$ as the solution of a generalized stochastic differential equation.

We are interested in infinite particle systems defined on stochastic flows. Basically, particles are dropped into $\mathbb{R}^n$ according to some point process over time, each particle moves in $\mathbb{R}^n$ according to the action of the flow on its position, and each particle eventually dies according to a killing mechanism that depends on the particle's path. The main interest is on the random measure valued process (M_t) where $M_t(A)$ is the total mass of all the particles that are in $A \subset \mathbb{R}^n$ at time t.

For an example, we may consider the flow as that of groundwater and we may think of particles as pollutants that enter the system, move with the flow, and eventually decay and disappear. Then, $M_t(A)$ would be the amount of pollutants in the volume A at time t.

The process (M_t) is Markov and its dynamics can be expressed via an infinite system of stochastic differential equations. We have been working on these equations and are trying to figure out the limiting behavior of M_t as $t \longrightarrow \infty$.

This is a totally different class of infinite particle systems. It differs from hydrodynamic limit of finite-particle systems, because the number of particles here is never fixed and is always infinite. It differs from the infinite particle systems of Ising model, etc., because the interactions between the particles are longer range.

This is a sizable research program. It seemed best to direct the research at the larger problem, instead of attacking some small part of it. This research is in progress.

List of Enclosures

1. Stochastic comparison of semimartingale Hunt processes; by B. BASSAN, E. CINLAR, and M. SCARSINI.

2. A Markov renewal approach to an opportunistic replacement model in continuous time, by D. P. KROESE and E. CINLAR.

3. Probability Theory and Stochastic Processes.
   Chapter 1. Measure and integration
   2. Probability spaces
   3. Convergence
   4. Conditioning
   5. Poisson random measures
   6. Martingales